Q- A point charge q is placed between two infinitely large conducting parallel planes with distance d between them, such that it is at a distance x from one plane. Find the charges induced on each plane.

Solution:

The planes are infinitely large and their potentials may be considered as zero. It is difficult to calculate the charges induced on the two planes using image method because in that case we have to consider a series of images, so here we use superposition methods.

The total flux emerging from the charge q is terminating on the two plates and accordingly inducing negative charges q_1 and q_2 respectively. Using superposition principle, we can say that the induced charges are same in quantity if the charge q is distributed to the imaginary plane P placed parallel to the two planes. Let the charge density at P is σ, and that induced on 1 and 2 is $-\sigma_1$ and $-\sigma_2$ respectively, then the electric fields $E_1 = \sigma_1/\varepsilon_0$ and $E_2 = \sigma_2/\varepsilon_0$

As the two planes has zero potential and if potential of P is V then $V = \sigma_1 x/\varepsilon_0 = \sigma_2(d-x)/\varepsilon_0$ gives

$$\sigma_1/\sigma_2 = (d-x)/x$$

As

$$\sigma_1 + \sigma_2 = \sigma,$$

We get $\sigma_1 = \sigma(d-x)/d$ and $\sigma_2 = \sigma x/d$

The induced charges are in the same ratio hence

$q_1 = \mathbf{-\,q(d-x)/d}$ and $q_2 = \mathbf{-\,q\,x/d}$

The actual effects of the induced charges is same as point charges $-q_1$ and $-q_2$ are placed at a distance x and d-x from the planes 1 and 2 on opposite side and the system behaves as two dipoles with charges q_1 and q_2 as in fig.