Reading:

Two particles are colliding head-on.

Let the masses of the particles be m_1 and m_2 and their initial velocities (along line joining) are u_1 and u_2 respectively (u_1 must be > u_2 for collision), and after collision moving with velocities v_1 and v_2 respectively ($v_2 > v_1$; separation)

1) If the collision is elastic. According to law of conservation of linear momentum and of kinetic energy

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad \text{………… (1)}$$

And $(1/2)\, m_1u_1^2 + (1/2)\, m_2u_2^2 = (1/2)\, m_1v_1^2 + (1/2)\, m_2v_2^2$,

Gives $m_1u_1^2 + m_2u_2^2 = m_1v_1^2 + m_2v_2^2$ ………… (2)

To avoid square terms, we can solve the equations for final velocities in this way: rearranging the equations we get

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2 \qquad \text{………. (1)}$$

And $m_1u_1^2 - m_1v_1^2 = m_2v_2^2 - m_2u_2^2$ ……. (2)

Dividing equation (2) by equation (1) and simplifying we get

$$u_1 + v_1 = u_2 + v_2$$

or $u_1 - u_2$ (velocity of approach) $= v_2 - v_1$ (velocity of separation) …. (3)

Using equation (1) and (3) is easier to solve for v_1 and v_2.

2) Now suppose that the collision is **inelastic**, here we define a new quantity, "coefficient of restitution" denoted by e which is the ratio of velocity of separation to the velocity of approach and equation (3) can be written as

$$e\,(u_1 - u_2) = (v_2 - v_1)$$

Clearly for elastic collision e = 1, for perfectly inelastic collision, the two bodies stick together and moves with the same velocity and hence e = 0. For rest of the collision e will have value between 0 and 1.

Hence in any head-on collision, for velocities of the particles after collision we have to solve the equations

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2 \qquad \text{………. (A)}$$

and $e\,(u_1 - u_2) = (v_2 - v_1)$ ………. (B)