Q- A wheel 1.9 m in diameter lies in a vertical plane and rotates about its central axis with a constant angular acceleration of 4.15 rad/s^2. The wheel starts from rest at t = 0, and the radius vector of a certain point P on the rim makes an angle of 57.3° with the horizontal at this time. At t = 2.00 s, find the following.

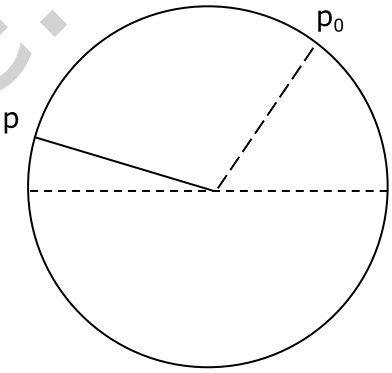
a) The angular speed to the wheel
b) The tangential speed of the point P
c) The total acceleration of the point P (magnitude and direction)
d) The angular position of the point P

Reading:
The problems of circular and rotational motion can be solved easily if you compare it with the concepts of translational motion and write the equations for the rotational motion with corresponding quantities in translational motion like

$\omega = \omega_o + \alpha t$	$(v = u + at)$
$\theta = \omega_o t + \frac{1}{2} \alpha t^2$	$(s = ut + \frac{1}{2} at^2)$
$\omega^2 = \omega_o^2 + 2\alpha\theta$	$(v^2 = u^2 + 2as)$
$\tau = I \alpha$	$(F = ma)$

With this you can relate with the quantities with that of translational motion with

$s = \theta R$
$v = \omega R$
$a = \alpha R$



Answer:

a) The angular speed of the wheel at t = 2.00 s is give by

$\omega = \omega_o + \alpha t$

Or $\omega = 0 + 4.15*2.00 =$ **8.30 rad/s**

b) The tangential speed of point P is given by

$v = \omega R$

or $v = 8.30*(1.9/2) =$ **7.86 m/s**

c) The point P have two accelerations, one is the tangential acceleration responsible to change its speed given by αR and the other is centripetal acceleration responsible to change the direction of motion called centripetal or radial acceleration given by $\omega^2 R$. as both the accelerations are perpendicular to each other the magnitude of the total acceleration is given by

$$a_{Total} = \sqrt{a_{tangential}{}^2 + a_{radial}{}^2}$$

Or $a_{Total} = \sqrt{(\alpha R)^2 + (\omega^2 R)^2}$

Or $a_{Total} = \sqrt{(4.15 * 0.95)^2 + (8.30^2 * 0.95)^2} = \sqrt{15.54 + 4283} = 65.56\, m/s^2$

And its direction is given by

$$\tan\theta = \frac{a_{radial}}{a_{translational}} = 16.6$$

Or $\theta = 86.55$ deg towards center from the tangent at P.

d) The angle turned by the point P in 2.00 s is given by the second equation as

$$\theta = \omega_o t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2} 4.15 * 4.00 = 8.30\, rad$$

Thus the new position of P will be 1 +8.30 = 9.30 rad = 532.89° …..(57.3 deg = 1 rad)

Thus, the position of P at t = 2.00 s is at 172.89° after one rotation of 360°.