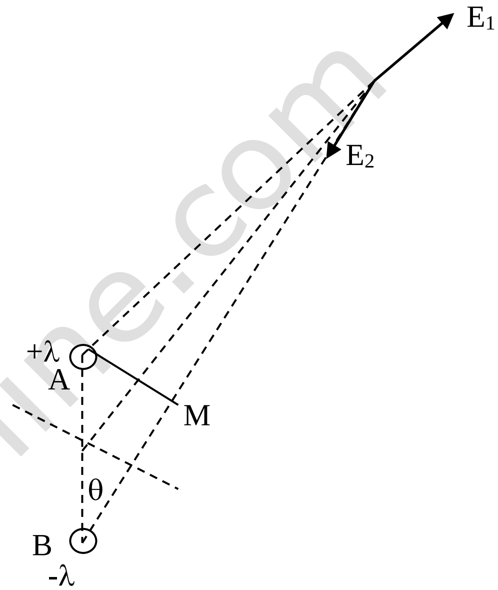
Q- Two thin long parallel threads carry's uniform charge density λ and -λ. The distance between them is s. Find the magnitude of the electric field E at a point P located at a distance r (such that r >> s) and making an angle θ with the vector s.

The charge on the wires per unit length is λ and – λ.

The electric field at distance r from a long straight wire having linear charge density λ is given by

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\,\hat{r}$$



Where $\hat{r}$ is unit vector in the direction of r.

The direction of the field is along the line joining the wire and the point P

As point P is at very large distance r the arc AM from P is near perpendicular and the thus the distance BM is given by s cos θ and hence we can say that the distance of positively charged wire from P is r - (s/2) cos θ and that from the other wire is r+ (s/2) sin θ.

Thus, the magnitude of the field due to positively charged wire at P will be

$$E_1 = \frac{\lambda}{2\pi\epsilon_0\left(r-\frac{s}{2}\cos\theta\right)}$$

And that due to negatively charged wire will be

$$E_2 = \frac{\lambda}{2\pi\epsilon_0\left(r+\frac{s}{2}\cos\theta\right)}$$

Now as the distance r is very large then the angle between the fields due to the two wires is nearly 180 degree and hence the fields are almost opposite. Thus, the resultant field is given by

$$E = E_1 - E_2 = \frac{\lambda}{2\pi\epsilon_0\left(r-\frac{s}{2}cos\,\theta\right)} - \frac{\lambda}{2\pi\epsilon_0\left(r+\frac{s}{2}cos\,\theta\right)}$$

Or $$E = \frac{\lambda}{2\pi\epsilon_0}\left(\frac{1}{\left(r-\frac{s}{2}cos\,\theta\right)} - \frac{1}{\left(r+\frac{s}{2}cos\,\theta\right)}\right)$$

Or $$E = \frac{\lambda}{2\pi\epsilon_0}\frac{\left(r+\frac{s}{2}cos\,\theta\right)-\left(r-\frac{s}{2}cos\,\theta\right)}{\left(r-\frac{s}{2}cos\,\theta\right)\left(r+\frac{s}{2}cos\,\theta\right)}$$

Or $$E = \frac{\lambda}{2\pi\epsilon_0}\frac{s\ cos\,\theta}{\left(r^2-\frac{s^2}{4}cos^2\theta\right)}$$

Again, the terms with s^2 can be neglected as compared to r^2 and we get.

$$E = \frac{\lambda}{2\pi\epsilon_0}\frac{s\ cos\,\theta}{r^2}$$

Hence the magnitude of the field due to both wires (at large distances) can be given by

$$E = \frac{\lambda s\ cos\,\theta}{2\pi\epsilon_0 r^{\ 2}}$$