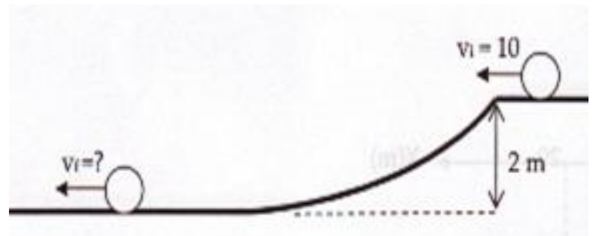
Q- A solid ball with mass M and radius R is rolling without slipping on a flat surface at 10 m/s. It then gets a small slope and rolls down 2 m from the top. Find the velocity of the ball after it passes the slope.

Solution:

When a sphere rolls without sliding on a plane surface the motion is a combination of translational and rotational motion. The velocity of centre of mass of the sphere v and the angular velocity about the axis through the centre w are related as



$v = \omega R$ ----------------- (1)

Now the total kinetic energy of the Rolling sphere is the sum of the translational kinetic energy ½ Mv^2 and the rotational kinetic energy ½ $I\omega^2$, thus

Total kinetic energy of rolling sphere K = ½ Mv^2 + ½ $I\omega^2$

Or $K = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{5}MR^2\omega^2$

Substituting the value from equation (1) we get

$K = \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2 = \frac{7}{10}Mv^2$ ---------------------------- (2)

Now as the ball crosses the hill, as there is no work done against any nonconservative force, according to law of conservation of mechanical energy the total mechanical energy of the ball will remains the same and hence

Gain in kinetic energy = loss in gravitational potential energy

Or final KE – initial KE = loss in PE

Or $\frac{7}{10}Mv_f^2 - \frac{7}{10}Mv_i^2 = Mgh$

Or $\frac{7}{10}\left(v_f^2 - v_i^2\right) = gh$

Gives $v_f^2 = v_i^2 + \frac{10}{7}gh$

Or $v_f = \sqrt{v_i^2 + \frac{10}{7}gh} = \sqrt{100 + \frac{10}{7} * 9.8 * 2} = 11.31\ m/s$