Q- A radio transmitter on the Earth's surface radiates a sinusoidal wave with an average total power of 50 kW. Assuming that the transmitter radiates equally in all directions above the ground calculate the amplitudes $E_{max}$ and $B_{max}$ detected by a satellite positioned 100 km from the transmitter.

For an electromagnetic wave Poynting vector s is defined as

$$\vec{S} = \frac{1}{\mu_0}\left(\overrightarrow{E_m} \times \overrightarrow{B_m}\right)$$

This vector is representing the energy flux i.e. the energy transported per unit area per unit time (called intensity of the wave also). Thus its units will be $J/m^2.s$ or $W/m^2$.

As the intensity of a wave is the amount of energy incident per unit area per unit time, the intensity of the wave at a distance r from a source of power P is given by (above the earth surface thus half sphere)

$$s = \frac{P}{2\pi r^2} \qquad \text{------------------ (1)}$$

As from the definition of Poynting vector the intensity (magnitude of Poynting vector) is given by (both fields are perpendicular to each other)

$$s = \frac{E_m * B_m}{\mu_0} = \frac{c * B_m * B_m}{\mu_0} = \frac{c\, B_m^2}{\mu_0} \qquad \text{------------------ (2)}$$

From equations (1) and (2) we get

$$\frac{c\, B_m^2}{\mu_0} = \frac{P}{2\pi r^2}$$

Gives $B_m = \sqrt{\frac{\mu_0 P}{2\pi r^2 c}} = \sqrt{\frac{2*10^{-7}*50*10^3}{(100*10^3)^2*3*10^8}} = 5.77 * 10^{-11}\, T$

And the amplitude of the electric field will be

$E_m = c*B_m = 3*10^8*5.77*10^{-11} = 1.72*10^{-2}$ N/C