Q- A steel shaft (density = 8050 kg/m^3) is 2m long and is accelerated from rest to 400 rpm in 6 seconds by a torque of 100 Nm. Determine the max diameter of the shaft.

Let the radius of the shaft be R
Length L (= 2m)
Density of steel ρ = 8050 kg/m^3
The volume of the shaft = $\pi R^2 * L$
And hence its mass will be $m = \pi R^2 * L * \rho$
Now the moment of inertia of the shaft (cylindrical) is given by

$$I = \frac{1}{2} m R^2 = \frac{1}{2}\left(\pi R^2 L \rho\right) R^2 = \frac{1}{2}\left(\pi R^4 L \rho\right) \quad \text{--------------- (1)}$$

Now
Initial angular velocity of the shaft $\omega_0 = 0$
Final angular velocity of the shaft ω = 400 rpm = 400*2π/60 = 40 π/3 radians/s
Time interval Δt = 6 sec.
Hence the angular acceleration required is given by

$$\alpha = \frac{\Delta \omega}{\Delta t} = \frac{\omega - \omega_0}{\Delta t} = \frac{40\pi/3}{6} = \frac{20\pi}{9} \text{ rad/s}^2 \quad \text{--------------- (2)}$$

Now like the equation of translational motion [F = ma] we can write the equation for rotational motion for the shaft as
Torque = Moment of inertia*angular acceleration
Or $\tau = I * \alpha$

Using equation 1 and 2 the equation transforms to

$$\tau = \frac{1}{2}\left(\pi R^4 L \rho\right) * \frac{20\pi}{9}$$

Gives $R = \left(\frac{9\tau}{10\pi^2 L \rho}\right)^{\frac{1}{4}}$

Substituting the numerical values, we have

$$R = \left(\frac{9*100}{10*3.14^2*2.0*8050}\right)^{\frac{1}{4}} = (5.67*10^{-4})^{\frac{1}{4}} = 0.154\, m$$

Hence the diameter of the shaft = 2R = 2*0.154 = 0.308 m = 30.8 cm

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