Q- The switch for the network of Figure has been closed for long time. It is then opened at the time defined as t = 0s.

(a) Determine the time required for the current i_R to drop to 1 mA.

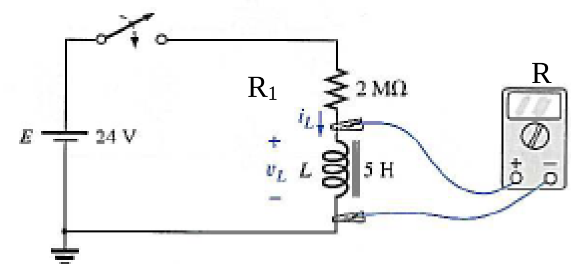
(b) Find the voltage v_L at t = 1 ms.

(c) Calculate v_{R_3} at t = 5 time constants.

When the switch is closed, this is a long time to current to become steady in the circuit. As the current is study there will be no potential different across the inductor as it is conducting and its resistance is negligible. Thus the current through the inductor is given by

$I_0 = E/R_1 = 24/(2*10^6)$

Or $I_0 = 12\ \mu A$



Now as the switch is opened at t = 0 the battery circuit get disconnected and the current through R_1 becomes zero.

With this the current in the inductor tends to zero but the decrease in the current in the coil develops an induced EMF in the coil and this EMF will flow the current in circuit available through the meter with resistance R. The magnetic field energy stored in the inductor during the growth of current through it is used to flow current through R and hence the energy and the current decrease with time.

The induced EMF e is the coil is given by

$e = - L\,(dI/dt)$

and hence using loop law we get for the closed circuit of inductor and the meter as

$- L\,(dI/dt) - IR = 0$

Gives $\frac{dI}{I} = -\frac{R}{L}dt$

Integrating above equation with the proper the current I as a function of time is given as

$$\int_{I0}^{I} \frac{dI}{I} = -\frac{R}{L}\int_0^t dt$$

Or $ln\left(\frac{I}{I_0}\right) = -\frac{Rt}{L}$

Or $I = I_0 * e^{-\frac{Rt}{L}}$ ---------------------- (1)

As the initial current $I_0 = 12\ \mu A$ the current at time t is given by

$$I = 12 * 10^{-6} * e^{-\frac{10*10^6*t}{5}}$$

And for this current to be 1 micro Ampere we have

$$1 * 10^{-6} = 12 * 10^{-6} * e^{-\frac{10*10^6*t}{5}}$$

Or $e^{-\frac{10*10^6*t}{5}} = \frac{1}{12}$

Or $2 * 10^6 * t = -\ ln\left(\frac{1}{12}\right)$

Gives t = 2.485/(2*10^6) = 1.243*10^{-6} s = **1.243 micro seconds**.

(b) The voltage across the inductor is the same across the resistor R and hence given by

$$V_L = IR = I_0R * e^{-\frac{Rt}{L}} \qquad \text{---------------------------- (2)}$$

Hence after one millisecond the voltage is given by

$$V_L = 12 * 10^{-6} * 10 * 10^6 * e^{-\frac{10*10^6*1*10^{-6}}{5}}$$

Or $V_L = 120 * e^{-\frac{10*10^6*1*10^{-6}}{5}}$

Gives **V_L = 16.24 V**

(c)

From equation (1) we can see that the quantity L/R in the power of e must have dimensions of time and hence this ratio is called time constant of the circuit and is denoted by τ.

Thus the equation (2) for the voltage across the inductor of across the resistor R can be written as

$$V_L = IR = I_0R * e^{-\frac{t}{\tau}}$$

And hence the voltage across the inductor or R at time t = 5τ is given by

$$V_R = 120 * e^{-\frac{5\tau}{\tau}} = 0.801\,V$$