Q- A heavy piece of hanging sculpture is suspended by a 90 cm long, 5.0 g steel wire. When the wind blows hard, the wire hums at its fundamental frequency of 80 Hz. What is the mass of the sculpture?

The frequency f, wave velocity c and the wavelength λ for a wave are related as

$$c = f * \lambda \qquad \text{--------------- (1)}$$

The velocity of a transverse wave on a stretched string is given by

$$c = \sqrt{\frac{T}{\mu}} \qquad \text{--------------- (2)}$$

Here T is the tension in the string and μ is the mass per unit length of the string.
Substituting value of c from equation 1 in equation 2 we get

$$f\lambda = \sqrt{\frac{T}{\mu}}$$

Or $$f = \frac{1}{\lambda}\sqrt{\frac{T}{\mu}} \qquad \text{--------------- (3)}$$

Now when a stretched wire oscillates with fundamental frequency, only one antinode in the middle is formed and the length of the two nodes formed at the ends of the wire are at a distance of λ/2. Thus in this situation (for fundamental oscillations) λ= 2L. The equation (3) can be written as

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Nodes are the points of minimum amplitude (stationary points) and the distance between two nearest nodes is also λ/2.

Now according to the question

| | |
|---|---|
| Fundamental frequency | f = 80 Hz |
| Mass of wire | m = 5.0 g = 0.005 kg |
| Length of string | L = 90 cm = 0.9 m |

Hence mass per unit length of the string

μ = m/L = 0.005/0.9 = (1/180) kg/m

Substituting data in above equation we get

$$80 = \frac{1}{2*0.9}\sqrt{\frac{T*180}{1}}$$

Gives $\sqrt{180\,T} = 80 * 2 * 0.9$

Or $\sqrt{180\,T} = 144$

Squaring both sides, we get the tension in the string as

$$T = 144^2 * \frac{5}{900} = 115.2\,N$$

As this tension is due to the weight of the hanging mass M we get

Mg = T = 115.2

Or M = 115.2/9.8 = **11.76 kg**.